ANALYTICAL AND MATHEMATICAL COMPETENCE DEVELOPMENT ACROSS LIFESPAN

Lena Lindenskov

Danish School of Education, Aarhus University – Copenhagen Campus, Denmark

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RN5: EVENTS

- 2008 Beijing ASEM Conference
- 2009 Copenhagen Network meeting
- 2009 Seoul Network Conference
- 2010 Copenhagen Coordination meeting
- 2010 Glasgow Network meeting (cancelled)
- 2010 Hanoi ASEM Conference
RN5 is expanding

- Korea
- Thailand
- Malaysia
- Japan
- Hong Kong
- Lithuania
- Great Britain
- Latvia
- Denmark
- Germany
- The Netherlands
RN 5 ACTIVITIES AND PRODUCTS

- Presentations of participants’ research
- History of competence constructs nationally and regionally
- Empirical investigations on analytical and mathematical competences

Products
- 3 papers in *Framework for supporting lifelong learning* (Beijing conference)
- 10 papers in *Managing and Developing Core Competences in a Learning Society* (Seoul conference)
- Comparative Research plan for analytical and mathematical competences across lifespan among lower secondary school pupils and among pensioners active in voluntary work
AIMS OF RN5 ON CORE COMPETENCES

- To analyse and develop theoretical constructs on core competences
  - so that politics and practice can be reflected and supported in critical constructive ways.
Aims of RN5 on Core Competences

- (to contribute) to more and more competent workforce supply, in both skills and attitudes terms—human capital and social capital as well as good citizenship—at the national, regional and global level.

(Patcharawalai Wongboonsin and Kua Wongboonsin on universities’ contribution)

by

- theoretical discussions in cooperation and comparison

- and empirical investigations

- accompanied with illustrative examples
CORE COMPETENCE ILLUSTRATED BY EXAMPLES

- Personal competence
- Managerial competence
- ..
- ..
- Personal and social competences
- Communicative competences
- Analytical competences
  - Operate in formal-logic systems
  - Create models
  - Sociological imagination (Joern Skovsgaard, Arne Carlsen)
RN5 DEVELOPS HOLISTIC VIEWS, MEANING

- Competences, each with
  - Cognitive aspects (knowledge)
  - Functional aspects (skills)
  - Social aspects (attitudes)

- Avoiding long checklists

- Try to avoid dogmatism
WHY CORE C INSTEAD OF KEY C

- A shorter list
- More holistic view of cognitive, functional, and social aspects
- Metaphorically
  - Key
  - Core
- Expanding business context competitive use of the core metaphor
**Key DeSeCo**

- Contribute to valued outcomes for societies and individuals;
- Help individuals **meet important demands** in a wide variety of contexts; and
- Be important not just for specialists but for all individuals.

**Core ASEM RN5**

- ....same....
- Help individuals **contribute to creatively formulating and solving challenges** in a wide variety of contexts; and
- ..... same ....
- Stress the involvement and development of identity of persons, communities, companies
- Combine personal and creative competence with competences related to content subject competences, so recommendations for lifelong learning and teaching can be made.
1) Communication in the mother tongue;
2) Communication in foreign languages;
3) Mathematical competence and basic competences in science and technology;
4) Digital competence;
5) Learning to learn;
6) Social and civic competences;
7) Sense of initiative and entrepreneurship;
8) Cultural awareness and expression.

Act autonomously
Using tools interactively
Interact in heterogeneously groups
KEY COMPETENCE: MATH COMPETENCE

- Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts).

Core C ASEM rn5

- analytic, dialogic, cultural, global/multicultural, personal, social CC

- Meta core competences ?????

- Evidence-based, exemplified, empirical research

- Theoretically criticising if and how the competence transforms the self as the form of commoditized human ability (Han, SoongHee)
HOW TO LET INDIVIDUALITY AND SUBJECT KNOWLEDGE DWELL INTO/VITALISE CORE COMPETENCES?
EXEMPLARY BY COMPETENCES AND MATHEMATICS
- BECAUSE POPULAR IMAGES ARE CHALLENGING
AND BECAUSE OF GROWING CONSENSUS OF RELEVANCE OF MATHEMATICS

- In most European documents and European studies
- I.e. a GB study says that costs to numeracy difficulties only are estimated at up to £763 million each year for UK: *Competent numeracy would thus appear not only important in relation to employability and the economy, but also as a protective factor in maintaining social cohesion.*

- By KPMG Foundation 2006, revised in 2009
- [http://www.everychildachancetrust.org/counts/index.cfm](http://www.everychildachancetrust.org/counts/index.cfm)
Because mathematics traditionally is used as the first subject example in the process of developing theory

- Jean Piaget 1896-1980
- Jean Lave
WHAT IS RELEVANT TO ACOMPLISH?

- Calculate numbers or letters 2+3 \( a-b \) \( y = x(7x+4) \)
- Acting while counting, measuring, designing, locating, playing, explaining (Alan Bishop, 1988)
- Logical thinking, analytical reasoning
- PIACC
- PISA
- Skovsgaard, Carlsen on analytical competences
  - Operate in formal-logic systems
  - Create models
  - Develop and use sociological imagination
ACTING IN SITUATIONS
RELATIONSHIP MATHEMATICAL AND ANALYTICAL COMPETENCES?

Mathematical competences

- Mathematical knowledge
- Semantic
  - Sociology
  - Philosophy

Analytical competences
ANALYTICAL AND MATHEMATICAL C AS LEGITIMATE CANDIDATES FOR CORE COMPETENCES ARE EXPLORED IN GRADE 8

- Students’ positionings in grade 8
  - doing being an identity

- Students’ perceptions of learning mathematics as contributing to analytical competences

- Students’ performance in mathematics measured by teachers, national examination, PISA etc
ESSAYS AND INTERVIEWS ON

- What is mathematics
- What is mathematical thinking
- What do you do, when solving mathematical problems

(Vibeke Hetmar, DPU)
Asem rn5 preliminary results, Latvia, Denmark, (Malaysia)

- Best performing students
  - Mathematics is seeing phenomena and processes in the world as objects in distinct new ways
  - Listen to the music in mathematics
  - Systematically follow procedures
  - A kind of reflection to choose rules to find answers
  - Turning complex problems to simpler

- Other students
  - Mathematics is to understand what they mean
  - Calculations
  - Complicated
  - Difficult
  - Confusing
  - Boring
IN RELATION TO OTHER STUDIES

- Similar to PISA results
  - OECD Sept 2010

- Danish curricula
  - Primary and lower secondary
  - DPU Master in mathematics education

- Supplement PISA questionnaires with student essays and student interviews

- Supplement with potential teaching/learning approaches with new communication forms in classrooms
THE AIM

- Is not to construct more fine-grained ways of putting students into boxes in hierarchy

- But to support politics and praxis to promote learning and teaching approaches which give better opportunities to fulfill potentials for more students to learn to know, to do, to be, to work, to contribute to workplace and society, and to live together

- by supporting people within their situated roles and to transcend specific role identities to decontextualise core competences
Rn5 explores diverse settings and diverse actors

- Curriculum scholars and practitioners (Korea)
- Front-office personnel and their clients (Lithuania)
- Participants in graduate seminar course (Korea)
- Adult unskilled workers in formal educations (USA)
- Educational leaders (USA)
- Universities (Thailand)
- Adult learners of basic mathematics (7 EU-countries)
- Informal, latent nature of competences with time-difference (Scotland)

This afternoon

- Older adults (Lithuania)
- Individualised citizens (Thailand)
- Liberians and post office staff (Vietnam)
- Teachers (Korea)
- Subject – and metacognition – learners (Malaysia)
- Students in lower secondary (Latvia, Denmark)
- Concepts on study and learning in Confucianism (Japan)
THANK YOU

- Updates and new research plans, new book, and new open source journal
- ASEM website  www.dpu.dk/ASEM
HOPE AND BREATH

- Quite a lot of hope is put into the different conceptualisations of core competences. Development of “knowledge, abilities, skills, attitudes and values orientations that are being used for a long period of time and have the impact on comprehensive development of personality, professional flexibility and mobility.”
  - Lauzackas p. 182 Vaiva Zuzevičiūtė and Jūratė Paulauskaitė
- Concepts of core competences are investigated in a broad area of settings and with various actors
  - Students at grade 8 (Latvia, Denmark)
- Theory
Students’ interest and enjoyment in mathematics - PISA 2003

Diagram showing the interest and enjoyment of mathematics among students in various countries. The diagram includes a scale for scores and indicates which differences between countries are statistically significant at the 0.05 level.
Strongest interest and enjoyment
Weakest interest and enjoyment
TOMORROW:

- Curriculum scholars and practitioners (Korea)
Some present data and results on relevance and effects in a negative way, i.e.:

- Unemployment
- Imprisoned
- Non-participation in work-related training
- Lower earnings
- Costly special educational needs provision
- Truancy
- Exclusion from school

..estimate that the costs to the public purse ... are up to £2.4 billion every year. Costs to numeracy difficulties only ... are estimated at up to £763 million each year.
THE ABOVE COMES FROM A UK STUDY

“Competent numeracy would thus appear not only important in relation to employability and the economy, but also as a protective factor in maintaining social cohesion”

By KPMG Foundation 2006 and revised in 2009

http://www.everychildachancetrust.org/counts/index.cfm
WHAT CAN RESEARCH ADD TO POLICY-MAKING

- Analyse where are the problems in theory foundation and in practice
- Exemplify
- Try out

- Problem of context
- Problem of temporal dislocation
- Problem of subject knowledge
- *limit the utility of competencies as a tool for educational analysis and planning.*
Lauzackas 2005  Core competences

- knowledge, abilities, skills, attitudes and value orientations that are being used for a long period of time and have the impact on comprehensive development of personality, professional flexibility and mobility

- Competence always involve the performance of a metacognitive act, although it may be involved tacitly.
Adomaitiene et al. (2003) define competence as a physical or mental ability of a human being to carry out a certain action or activity. According to the number of authors and references, core competencies are identified as comprehensive and applicable within different spheres of life and professional activities. Furthermore, they play an important role in the context of constant economical and social, as well as technological changes. Core competencies (and their elements) are the result of lifelong learning.
There is a need for Vaiva and ......
The alternative is risky, too
Here it is important to note
that in this paper development of core competencies is at focus
(rather than learning in order to know how to operate new line
of technology better), because in labor market professional competence,
personal qualities and social competence have a crucial
importance today
Yet the solidity of the positivist warrant recedes towards a horizon of uncertainty as it is approached. Partly this is because competencies are the creation of a performative demand and a discursive formation rather than well-considered heuristics for knowledge. Partly, however, it is because competency systems are fundamentally incomplete and (in all senses of the word) partial. One aspect of this incompleteness is that competencies cannot capture everything. Any attempt to do so soon becomes very similar to the game children sometimes play where one takes on the role of a robot and the other gives instructions, which very soon have to be incredibly detailed and onerous. “Pick up the stick” becomes “reach out your left hand, move it down, open the fingers and the thumb” and so on. A degree of background knowledge is assumed for competencies, as pointed out by Hayek (1944). The boundaries of this tacit knowledge are the key to making sense of the concept, yet they fatally undermine the epistemological validity of competencies.

This discussion looks at one aspect of the incompleteness of competencies that has been not been well explored, that of temporal incompleteness. Many approaches to competency ignore the dimension of time, leaving competencies floating in some eternal present. This can lead, in turn, to the assumption that development and demonstration of competencies are seamlessly synchronous.
So when we ask questions about the effects of literacy learning, what we need to know is the potential outcomes in any given area. It is misleading to ask what the outcomes will be simply because people may have no need to situate their skills within that context. Instead, the focus must be on what capabilities can bring to that outcome when they are required and activated. The potential of literacy can only be understood when its expression and meaning within a given context is explored and the temptation to look for absolute statements is resisted. The same is true, I suggest, for competencies.
Demographic shift

One may think of the global pole will turn to Asia in years to come.

Yet, this study maintains that the prospects for Asia are not guaranteed, unless there is a proactive measure towards more competent and productive workforce. And, this leads to the role of an institution of higher learning, to turn itself into a research university, a hub of life-long education and training for the workforce, along university-industry partnership, with commitments to the international standards of academic activities, and an emphasis on the integration of research and instruction.
The more an institution of higher learning turning itself into a research university, the more it contribute to socialization of learning, where a learner watches and interacts with an expert/mentor/supervisor, which contributes to a whole body of experience. This is based on the notion by Delahaye and Becker (2006) that once the tacit knowledge is translated and expressed into forms that are comprehensible to the conscious mind of an individual and to others, the tacit knowledge is externalized into explicit knowledge.

In Europe, the Commission of the European Communities (2005) set out eight key competences for policy makers, education providers, employers, and learners themselves to facilitate national and European level effort towards personal fulfillment and development, active citizenship, social inclusion and employment. They are:

1. Communication in the mother tongue;
2. Communication in the foreign languages;
3. Mathematical competence and basic competences in science and technology;
4. Digital competence;
5. Learning to learn;
6. Interpersonal, intercultural and social competences and civic competence;
7. Entrepreneurship; and
8. Cultural expression.

Those key competences are based on the notion that by the end of initial education and training young people should have developed the key competences to a level that equips them for adult life, and they should be further developed, maintained and updated as part of lifelong learning. Many of the competences overlap and interlock. In other words, aspects essential to one domain will support competence in another. Competence in the fundamental basic skills of language, literacy, numeracy and ICT is an essential foundation for learning, and learning to learn supports all learning activities (Commission of the European Communities 2005).
Malaysia, Japan

To understand the shift we are in
THIS TALK CONTAINS:

- Some of my assumptions
- What in my view ought to be questioned
- Data and discussion on LLL and numeracy-mathematics
- Theoretical constructs
- Suggestions for practice and research
I PRESUME

- From a perspective of lifelong learning in a changing world nobody can assume to stop learning.

- Some of this later learning will include some mathematics.

- Still when it does not include any mathematics, experiences from learning and practising math may influence the lifelong learning.
I PRESUME

- Supporting people to continuously being capable and motivated to learn – with and without mathematics - will be a growing need
I PRESUME

- We will see in the future new roles for adult learning centres, including *Teaching/guiding adult learners to learn in practice*

www.statvoks.no/mia
I recognise a growing consensus on Math/Numeracy’s relevance for all.

In many reports being numerate or mathematical literate is seen as relevant for all and is illustrated by examples from everyday settings and educational and workplace settings.
In a new Danish study on Dyscalculia also negative stories are told

- Stories told of children, adolescents and adults experiencing failure in mathematics. STILL!
- Even stories of lowered general self-esteem and math anxiety are told. STILL!
This means contrasting societal discourses co-exist – they do not fit together

- Although the problems of relevance contrasting problems of performance, attitudes and feelings has been studied and discussed for decades
Therefore I keep thinking:

- Why not let the strength of learning mathematics strengthens all, instead of using mathematics learning - as a stop - as a screening examination - no admittance

- And instead of Daniel Sellers’ tartan maths ceiling
In my view it calls for renewed focus in the questions for research and practice

- For instance it calls for a question like:
  - How to make learning mathematics a door to lifelong learning for all?
- And it calls for investigating it by data and empirical research results
  - As well as by theoretical constructs
LET’S INVESTIGATE HOW IT MAY BE FRUITFUL, WHEN YOU DO NOT EXPERIENCE FAILURE AND ARE NOT EXCLUDED:

- Then HOW may it be fruitfull with economic benefit and with other benefits?
- And WHY is it so?
DATA, RESULTS ON RELEVANCE AND EFFECTS POSITIVELY DESCRIBED

- Helena Skyt Nielsen, Aarhus University found in her research (September 2009):

  33 % higher wage for students with highest level of mathematics 19 years old, entering university
ANSWERING THE WHY-QUESTION CALLS FOR RELATIONAL OR INDIVIDUAL CAUSES

- Individual causes include:
  - individual intelligence,
  - individual creativity,
  - or even disposition for authoritarian thinking

- Relational causes include:
  - relationship between (wo)man and mathematics

(Relational causes are what I myself find most appealing for further examination)
SEPTEMBER 2009 I WAS ASKED TO REFLECT ON POSSIBLE CAUSES:

- In my view the subject (mathematics / numeracy) may invite participants to develop
  - competences 'stuffed with creativity' in great demand,
  - ability to analyse, to recognise connections, and to move between abstract and concrete.

- Besides, experiencing you understands mathematics may provide important added bonus: including improved self-confidence and also the courage to learn more, which is so decisive.

- On the other hand, failure in mathematics can diminish self-confidence in mathematics and in other subjects, too. I cite Martin Wagenschein (1896 – 1988):

  The aim of teaching mathematics must be to contribute to pupils’ self-confidence, which no-one can live without. Thus pupils who do not always find mathematics easy to learn can be the most important for teaching mathematics, although this may not be the first coming to your mind.
RELATIONAL QUOTES ON EFFECTS OF PREPARATORY ADULT EDUCATION, PAE IN DENMARK

- Quotes from four adult participants
  A: Because I understand I feel good inside me and I got tasks I can solve. I learn a lot from others.
  B: It is best with many in class. All learn from each other – it feels good inside. When someone is better than me, then I talk and gets help in math.
  C: Different ages, different humour.
  D: I’ll definitely take more courses as a pensionist – in litterature I think.

- Quote from an enterprise:
  ARLA FOODS, DALOON:
  They [the workers/LL] are now able to present ideas of how to improve working processes in the enterprise orally and in writing.

Quotes are all from from Carter, J. (2004)
In my view learning mathematics may be a door to lifelong learning for all through the means of

- Contributing to analytical competences
- Contributing to learning in practice
- Contributing to ways of positioning as ’doing being’ a confident mathematical / analytical identity

Reference: J.P. Gee, 2001

SO it is not just a matter of having more or less anxiety
ANALYTICAL COMPETENCES ARE MATH-INCLUDING COMPETENCES:

- As *being a doer of mathematics*
- In a lifelong perspective - as part of the ASEM-project - the term *analytical competences* is described as one of three meta-competences and refers to a number of non-subject-specific abilities including the ability
  - to operate in formal-logical systems,
  - to create models
  - and to practice sociological imagination

In the ASEM HUB research we aim at:

- Develop theoretical constructs for core competences
  - In workplace
  - As responsible, critical citizens, regionally and globally
  - In family and other social relations

- Develop instructional models providing opportunities to learn better for more students

- www.dpu.dk/
A gr. 8 class with 19 students
**Oral questions. Written essays**

- What is mathematics?
- What is mathematical thinking?
- How to solve a mathematical problem
FIRST ANALYSIS STEP

- Highest performing: systematic, simplifying, thinking, careful planning, like music, intelligence
- Middle performing: logic, logical, rules, one right result, calculate, difficult, boring, intelligence
- Lowest performing: logic, logical, rules, one right result, calculate, difficult, boring, intelligence
SECOND ANALYSIS STEP

- High performers: M. seeing of things, thinking in units, reducing complexity, listening to the music in M
  - Position themselves as self-confident doers
- Low performers: intelligent people are capable, I am not
  - Mathematical thinking is to understand what they mean
  - Position themselves as engaged in discourse-for-others
THIRD ANALYSIS STEP

- Looking critically from the point of view of belief theory and positioning theory
  - Belief theory need to be developed as a framework fertilised with in-depth small-scale longitudinal investigations
  - Positioning theory sees identity as (re)constituted through various discursive practices, and concerns oral concersation, classroom communication, written products. We extend it to a basic analysis category for students’ statements
Answer: Learning mathematics can be a door to lifelong learning for all by

- Contributing to analytical competences

and

- Contributing to positioning as doing being a confident mathematical / analytical identity through effective activities and evaluations
A 'Persil task' like:

- How many grams do you get for 1p in the small size?
- How many grams do you get for 1p in the large size? (Remember you must work in grams and pence)
- Which size gives more for your money?


Dowlin: The school must foreclose on a privileged set of answers that maps onto its curriculum. In general, a mathematics curriculum must entail the privileging of mathematical principles.

In my view mathematical principles ARE indeed broader.
Tasks to be solved in school mathematics B C

- B is for training – expanding range, improving security, speed,

- C is for theoretical construction, as show – investigate – the area of a equilateral triangle with side length \( b \) is \( \frac{\sqrt{3}}{4} \times b^2 \)
Situations and phenomena to be discussed and handled in learning mathematics

- ALMAB
- EMMA
- MiA: organise learning in adult education like 'authentic' learning situations in life
- ..... 

Critique from Alan Schoenfeld 1985 towards A B C D
- 'let’s just get it done'
- and neglect reflection, communication, dwelling, enjoying

Potentials in A,B,C,D
ADULTS LEARNING MATHEMATICS – AIMS AND MEANS - AND RESEARCH QUESTIONS

- Positioning and communication
- Participating in lifelong learning
- ‘Simple’ development of A B C D
- Contribute to analytical competences
IN SOCIETY

Learning organised and financed to support

Participating in lifelong learning

Societal discourse on mathematics to support

Practice decides if the learnt is used
TWO QUESTIONS:

A. Did you yourself engage in education or training during the last four weeks?
B. If yes, was it work-related or not?
C. And if yes, did your mathematical competences and numeracy help to engage?

A       yes  no  don’t know
B       yes  no  don’t know
C       yes  no  don’t know
INVITATION

- Invite to further experimental research on teaching-learning activities and processes – with short and long terms perspectives - [www.dyscalculia.dk](http://www.dyscalculia.dk) [www.cormea.com](http://www.cormea.com)
- Invite to let positioning become important element of a framework for competence development in LLL-perspective.
- Invite to see new demands for competence as tightly interwoven with demands for new identity.

- And as always: Invite to reflect and handle the tension between humanistic and economic regimes in education
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- http://www.cormea.org
APPENDIX 2: AMOUNT OF LIFELONG LEARNING

- Measured by individuals reporting received education or training in the four weeks preceding survey for aged 25 to 64 in EU-countries 2008 [Eurostat]
APPENDIX 1: INTEREST, ENJOYMENT IN MATH PISA 2003